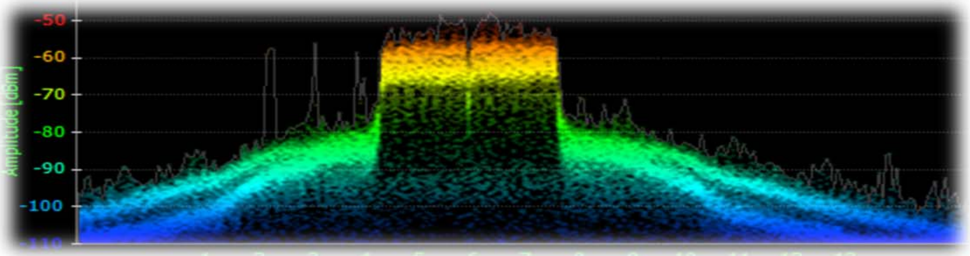

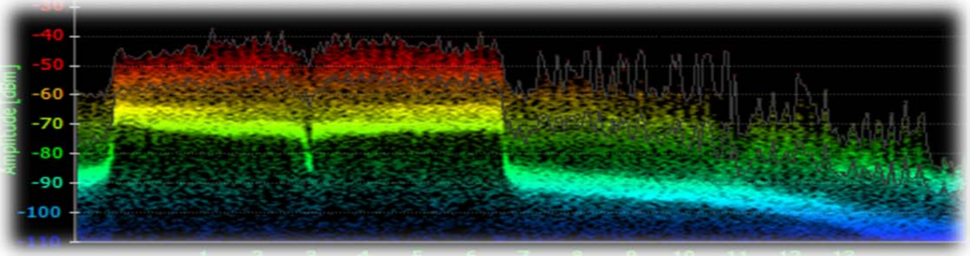

ECS455: Chapter 5

OFDM

5.2 Multi-Carrier Transmission

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Office Hours:
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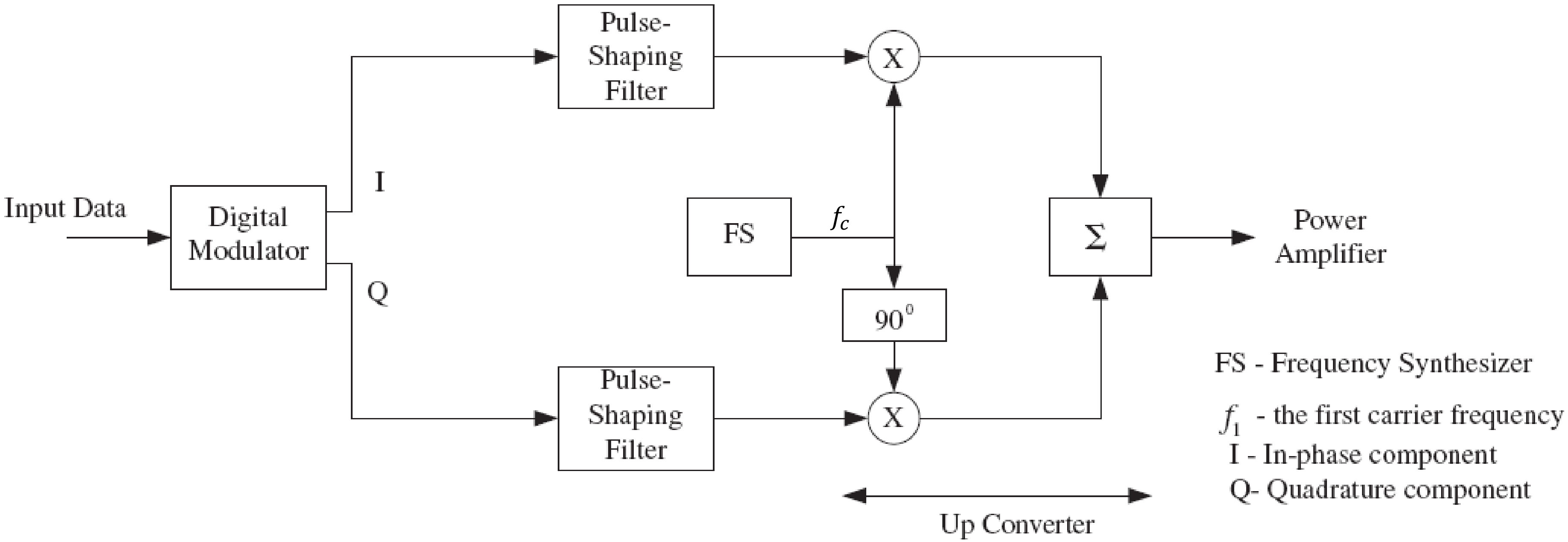
Tuesday	**9:30-10:30**
Tuesday	**13:30-14:30**
Thursday	**13:30-14:30**

Single-Carrier Transmission



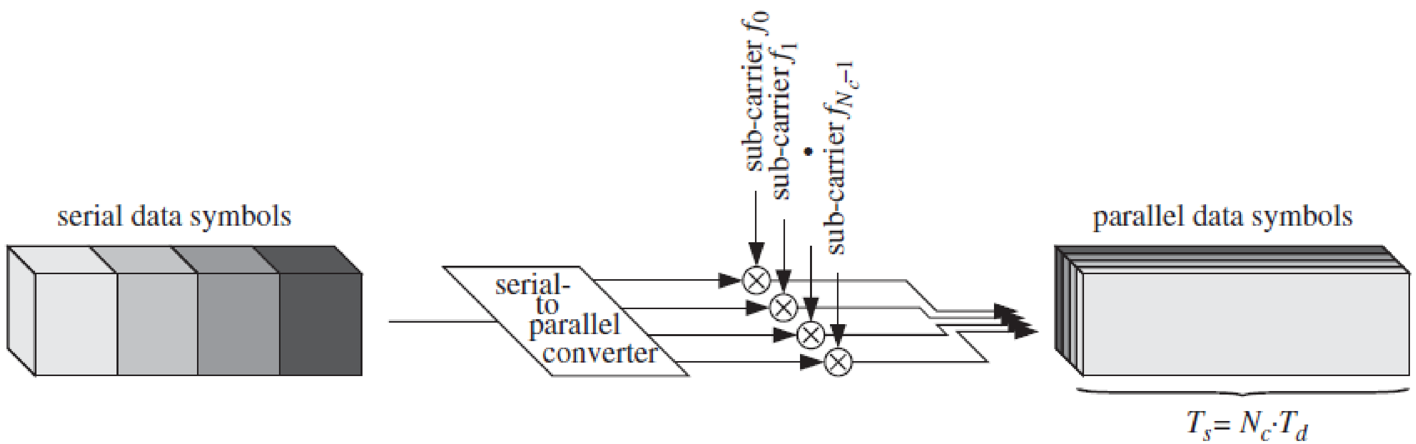
[Karim and Sarraf, 2002, Fig 3-1]

$$s(t) = \overbrace{S_I p_I(t)}^{\text{In-phase component}} \cos(\omega_c t) - \overbrace{S_Q p_Q(t)}^{\text{Quadrature component}} \sin(\omega_c t)$$

$$= \mathrm{Re}\left\{ \left(S_I p_I(t) + jS_Q p_Q(t) \right) e^{j\omega_c t} \right\}$$

Multi-Carrier Transmission

- Convert a serial high rate data stream on to **multiple parallel low rate** sub-streams.
- Each sub-stream is modulated on its own sub-carrier.
- **<u>Time domain perspective</u>**: Since the symbol rate on each sub-carrier is much less than the initial serial data symbol rate, the effects of delay spread, i.e. ISI, significantly decrease, reducing the complexity of the equalizer.

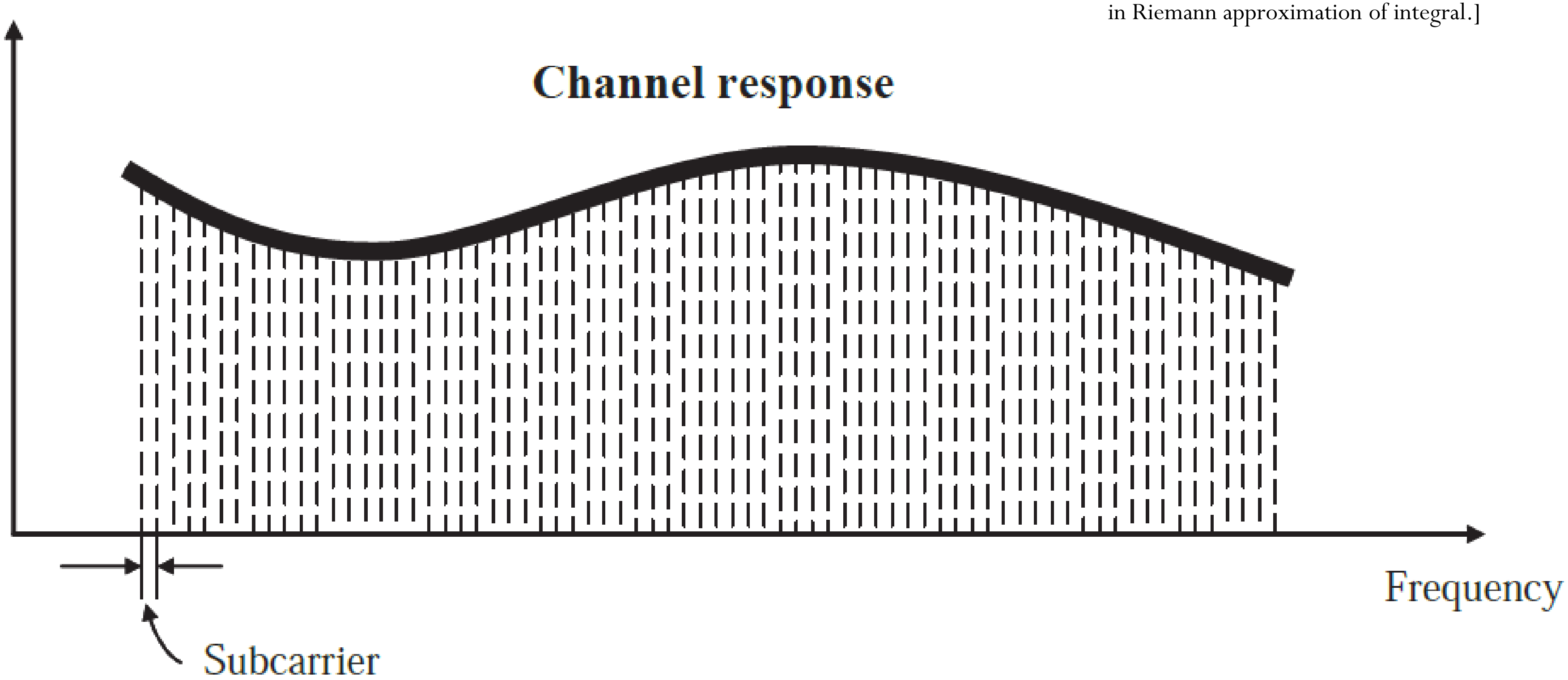


[Fazel and Kaiser, 2008, Fig 1-4]

Frequency Division Multiplexing

- **Frequency Domain Perspective**: Even though the fast fading is frequency-selective across the entire OFDM signal band, it is effectively flat in the band of each low-speed signal.

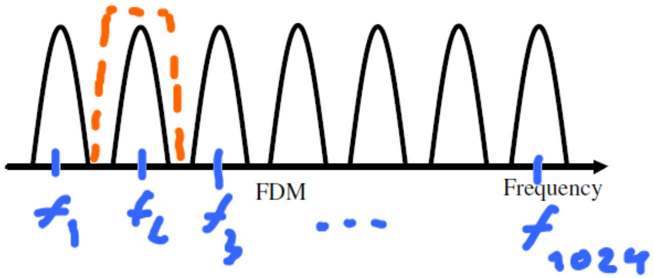
[The flatness assumption is the same one that you used in Riemann approximation of integral.]



[Myung and Goodman, 2008]

Frequency Division Multiplexing

- To facilitate separation of the signals at the receiver, the carrier frequencies were **spaced sufficiently far apart** so that the signal spectra did not overlap. Empty spectral regions between the signals assured that they could be separated with readily realizable filters.
- The resulting spectral efficiency was therefore quite low.



Multi-Carrier (FDM) vs. Single Carrier

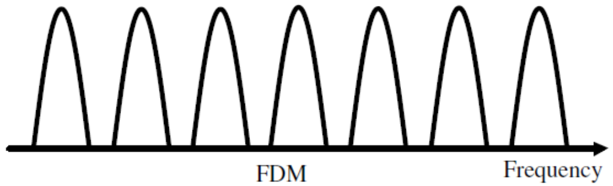
Single Carrier	Multi-Carrier (FDM)
Single higher rate serial scheme	Parallel scheme. Each of the parallel subchannels can carry a low signalling rate, proportional to its bandwidth.
❍ Multipath problem: Far more susceptible to inter-symbol interference (ISI) due to the short duration of its signal elements and the higher distortion produced by its wider frequency band ❍ Complicated equalization	❍ Long duration signal elements and narrow bandwidth in sub-channels. ❍ Complexity problem: If built straightforwardly as several (*N*) transmitters and receivers, will be more costly to implement. ❍ BW efficiency problem: The sum of parallel signalling rates is less than can be carried by a single serial channel of that combined bandwidth because of the unused guard space between the parallel sub-carriers.

FDM (con't)

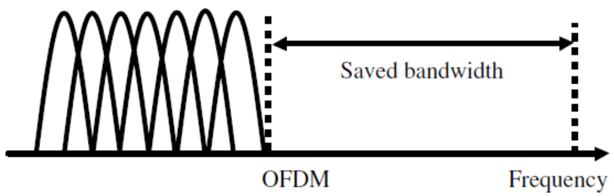
- Before the development of equalization, the parallel technique was the preferred means of achieving high rates over a dispersive channel, in spite of its high cost and relative bandwidth inefficiency.

OFDM

- OFDM = Orthogonal frequency division multiplexing
- One of multi-carrier modulation (MCM) techniques
  - Parallel data transmission (of many sequential streams)
  - A broadband is divided into many narrow sub-channels
  - Frequency division multiplexing (FDM)



- High spectral efficiency
  - The sub-channels are made orthogonal to each other over the OFDM symbol duration T_s.
    - Spacing is carefully selected.
  - Allow the sub-channels to overlap in the frequency domain.
  - Allow sub-carriers to be spaced as close as theoretically possible.



OFDM

- Recall: Orthogonality-Based MA (CDMA)

$$s(t) = \sum_{k=0}^{\ell-1} S_k c_k(t) \quad \text{where} \quad c_{k_1} \perp c_{k_2}$$

- Discrete baseband OFDM modulated symbol:

$$f_k = \frac{k}{T_s}$$

$$s(t) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} S_k \exp\left(j \frac{2\pi k t}{T_s} \right), \quad 0 \le t \le T_s$$

$$= \sum_{k=0}^{N-1} S_k \underbrace{\frac{1}{\sqrt{N}} 1_{[0,T_s]}(t) \exp\left(j \frac{2\pi k t}{T_s} \right)}_{c_k(t)}$$

Another special case of CDMA!

OFDM: Orthogonality

$$\int c_{k_1}(t)c^*_{k_2}(t)dt = \int_0^{T_s} \exp\left(j\frac{2\pi k_1 t}{T_s}\right)\exp\left(-j\frac{2\pi k_2 t}{T_s}\right)dt$$

$$= \int_0^{T_s} \exp\left(j\frac{2\pi(k_1-k_2)t}{T_s}\right)dt = \begin{cases} T_s, & k_1 = k_2 \\ 0, & k_1 \neq k_2 \end{cases}$$

When $k_1 = k_2$,

$$\int c_{k_1}(t)c^*_{k_2}(t)dt = \int_0^{T_s} 1dt = T_s$$

When $k_1 \neq k_2$,

$$\int c_{k_1}(t)c^*_{k_2}(t)dt = \frac{T_s}{j2\pi(k_1-k_2)}\exp\left(j\frac{2\pi(k_1-k_2)t}{T_s}\right)\Bigg|_0^{T_s}$$

$$= \frac{T_s}{j2\pi(k_1-k_2)}(1-1) = 0$$

Frequency Spectrum

$$s(t)=\sum_{k=0}^{N-1} S_k \underbrace{\frac{1}{\sqrt{N}} 1_{[0,T_s]}(t)\exp\left(j\frac{2\pi kt}{T_s}\right)}_{c_k(t)}$$

$$\Delta f = \frac{1}{T_s}$$

This is the term that makes the technique FDM.

$$1_{\left[-\frac{T_s}{2},\frac{T_s}{2}\right]}(t) \xrightarrow{\mathcal{F}} T_s \operatorname{sinc}(\pi T_s f)$$

$$c(t)=\frac{1}{\sqrt{N}}1_{[0,T_s]}(t) \xrightarrow{\mathcal{F}} C(f)=\frac{1}{\sqrt{N}}T_s e^{-j2\pi f\frac{T_s}{2}} \operatorname{sinc}(\pi T_s f)$$

$$c_k(t)=c(t)\exp\left(j\frac{2\pi kt}{T_s}\right) \xrightarrow{\mathcal{F}} C_k(f)=C\left(f-\frac{k}{T_s}\right)=C(f-k\Delta f)$$

$$s(t)=\sum_{k=0}^{N-1} S_k c_k(t) \xrightarrow{\mathcal{F}} S(f)=\sum_{k=0}^{N-1} S_k C_k(f)$$

$$=\frac{1}{\sqrt{N}}\sum_{k=0}^{N-1} S_k e^{-j2\pi(f-k\Delta f)\frac{T_s}{2}} T_s \operatorname{sinc}(\pi T_s(f-k\Delta f))$$

Subcarrier Spacing

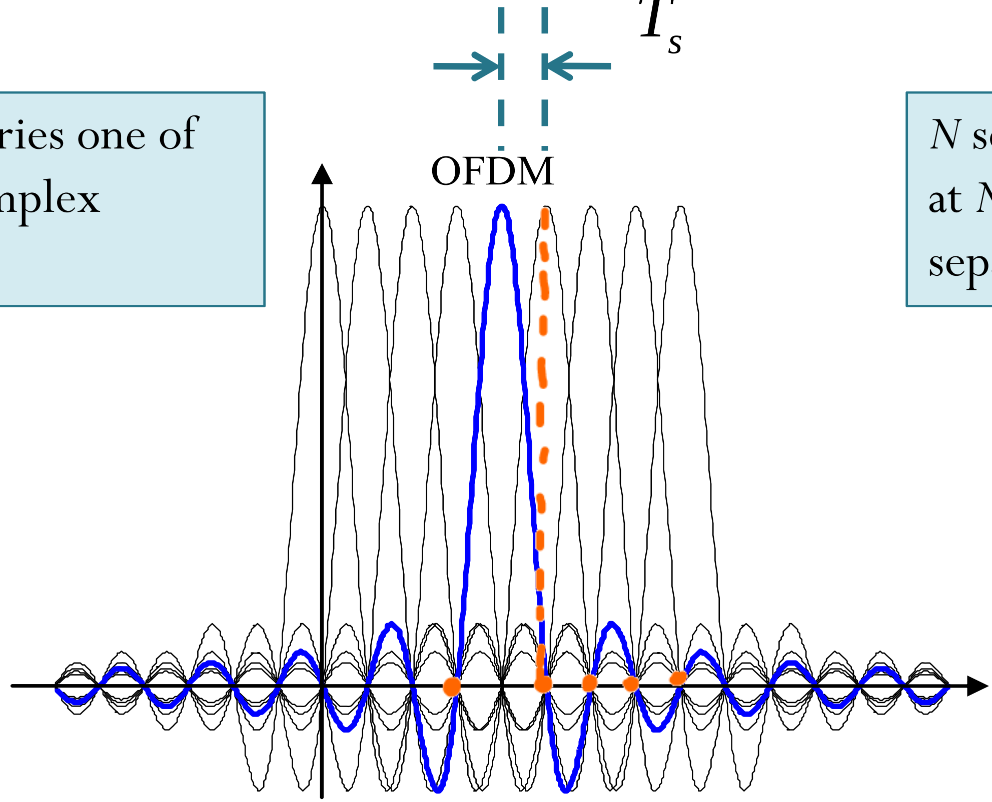
$$s(t) = \sum_{k=0}^{N-1} S_k \frac{1}{\sqrt{N}} 1_{[0,T_s]}(t) \exp\left(j\frac{2\pi kt}{T_s} \right)$$

$$S(f) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} S_k e^{-j2\pi(f-k\Delta f)\frac{T_s}{2}} T_s \operatorname{sinc}\left(\pi T_s (f - k\Delta f) \right)$$

$$\Delta f = \frac{1}{T_s}$$

Each QAM signal carries one of the original input complex numbers.

N separate QAM signals, at N frequencies separated by the signaling rate.

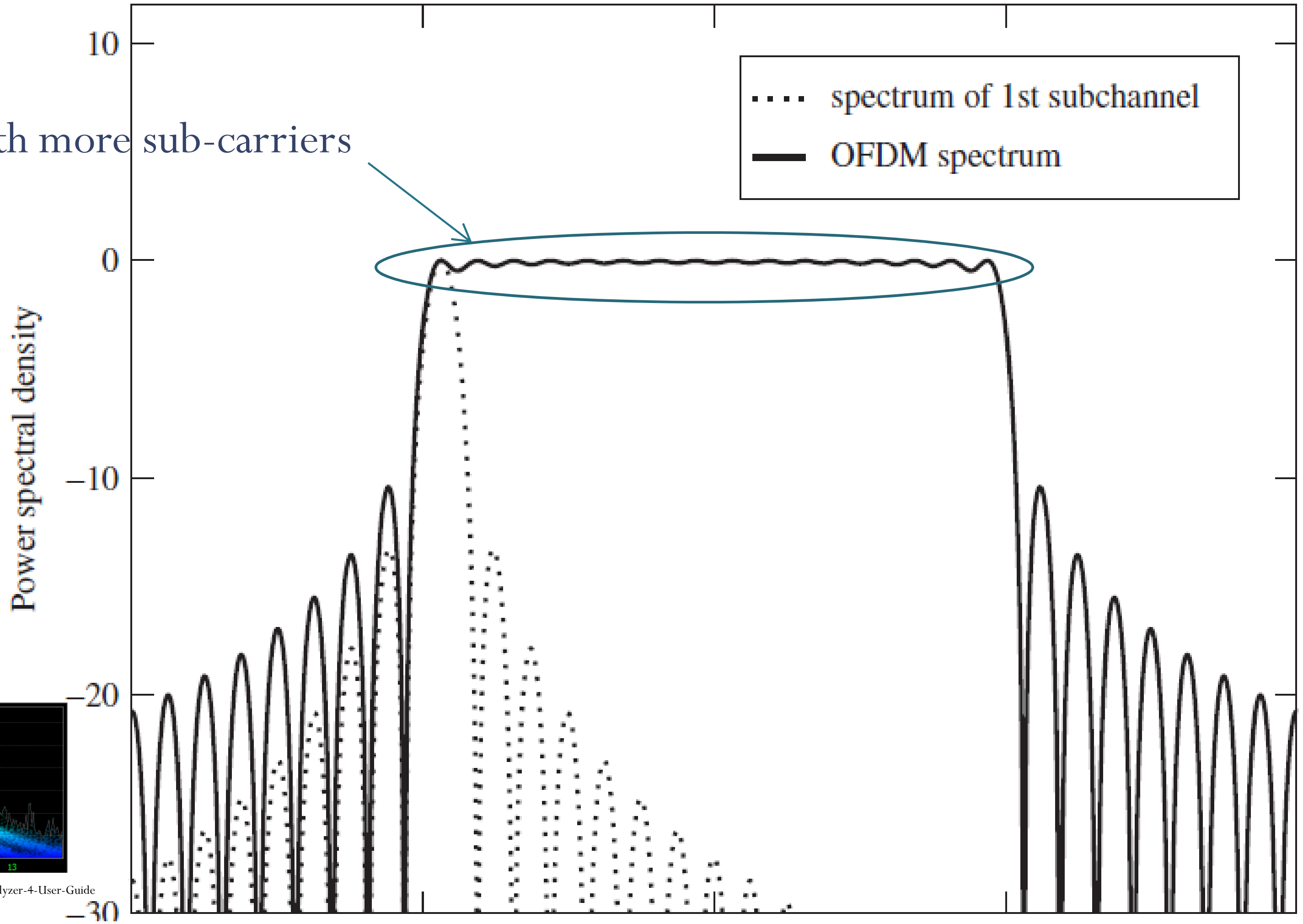


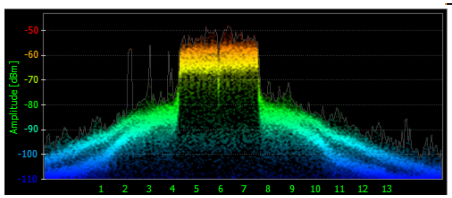
Spectrum Overlap in OFDM

The spectrum of each QAM signal is of the form with nulls at the center of the other sub-carriers.

Normalized Power Density Spectrum

More flat with more sub-carriers



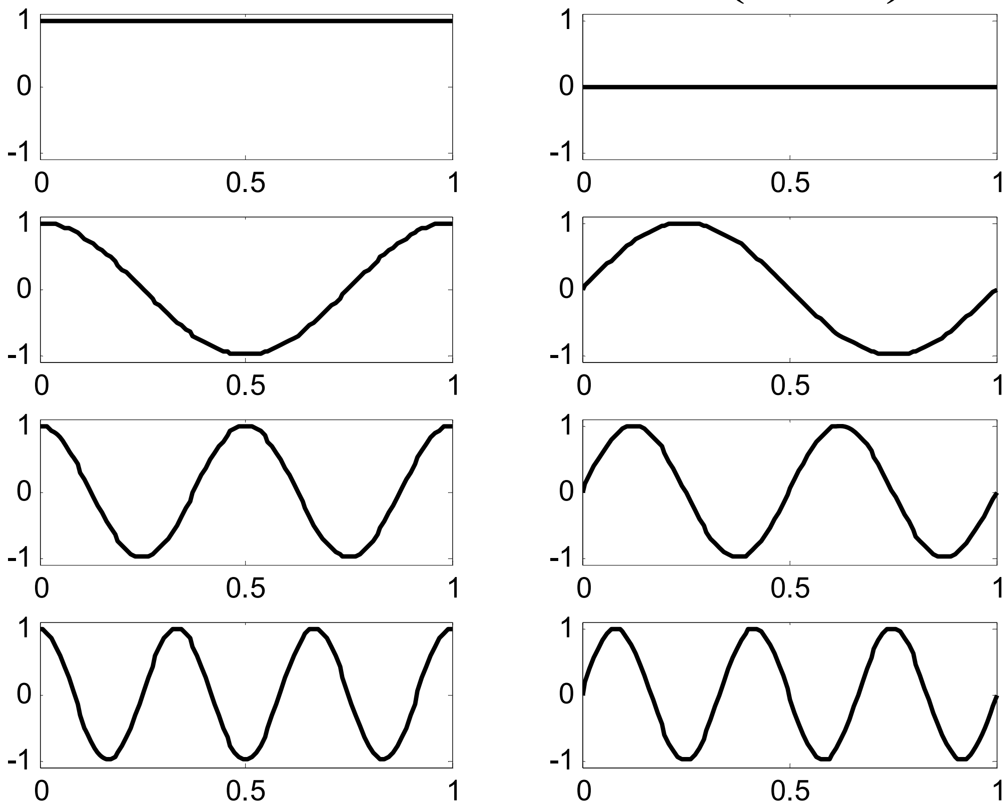


http://www.metageek.net/forums/showthread.php?4912-Chanalyzer-4-User-Guide

[Fazel and Kaiser, 2008, Fig 1-5]

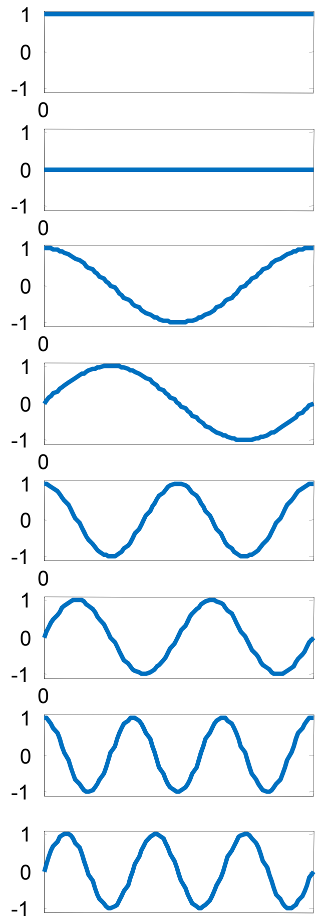
OFDM Carriers: N = 4

$$\cos\left(\frac{2\pi kt}{T_s}\right) \qquad \sin\left(\frac{2\pi kt}{T_s}\right)$$

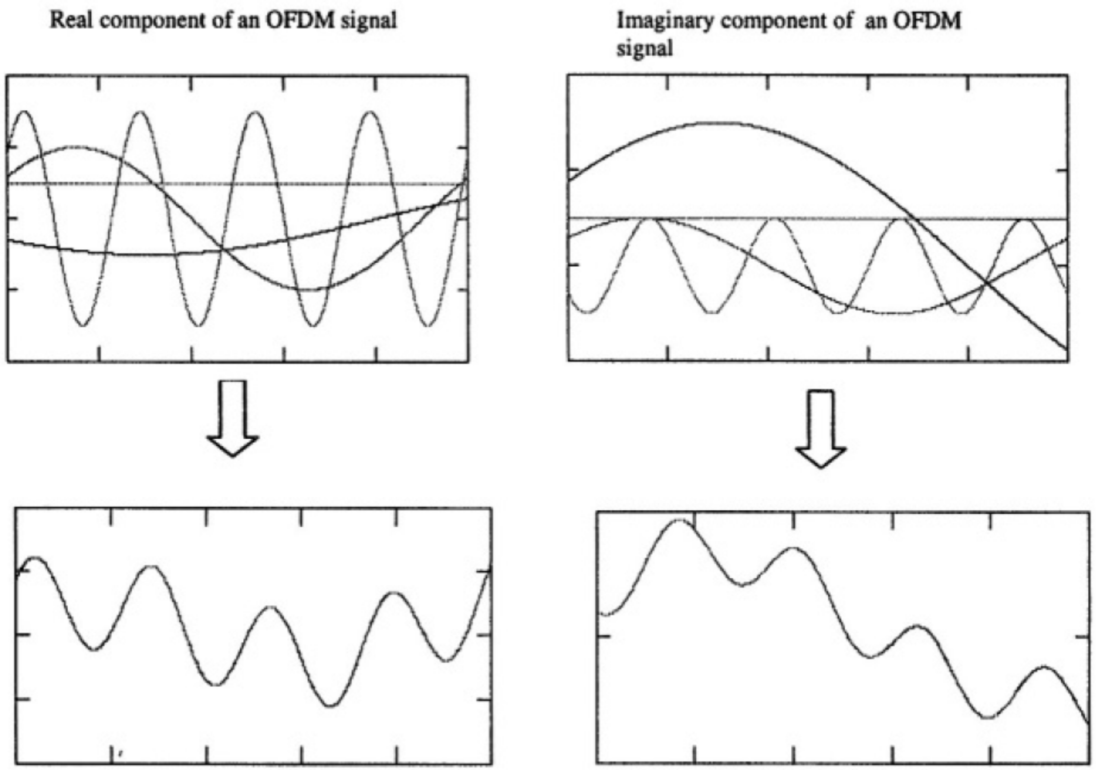


OFDM as a Multicarrier Technique

$$\mathrm{Re}\{s(t)\} = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} \left(\mathrm{Re}\{S_k\} \cos\left(\frac{2\pi k t}{T_s}\right) - \mathrm{Im}\{S_k\} \sin\left(\frac{2\pi k t}{T_s}\right) \right)$$



Time-Domain Signal



[Bahai, 2002, Fig 1.7]

Real and Imaginary components of an OFDM symbol is the *superposition of several harmonics* modulated by data symbols

$$s(t) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} S_k \exp\left(j \frac{2\pi k t}{T_s} \right), \quad 0 \le t \le T_s$$

$$\mathrm{Re}\{s(t)\} = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} \left(\underbrace{\mathrm{Re}\{S_k\} \cos\left(\frac{2\pi k t}{T_s} \right)}_{\text{in-phase part}} - \underbrace{\mathrm{Im}\{S_k\} \sin\left(\frac{2\pi k t}{T_s} \right)}_{\text{quadrature part}} \right)$$

Summary

- So, we have a scheme which achieves
  - Large symbol duration (T_s) and hence less multipath problem
  - Good spectral efficiency
- One more problem:
  - There are so many carriers!